CS 3310 fall 2020 final. Closed book, notes, computer, etc.   Name: Answers

1. Write the truth table for the proposition $\neg(r \to \neg q) \lor (p \land \neg r)$.

| $p$ | $q$ | $r$ | $\neg(r \to \neg q) \lor (p \land \neg r)$ / !(!r V !q) V (p ∧ !r) |
|---|---|---|---|
| T | T | T | T F T F |
| T | T | F | F T T T |
| T | F | T | F T F F |
| T | F | F | F T T T |
| F | T | T | T F T F |
| F | T | F | F T F F |
| F | F | T | F T F F |
| F | F | F | F T F F |

2. Write a proposition equivalent to $p \to q$ using only $p, q, \neg$ and the connective $\land$.

(!p V q)

!!(!p V q)

~~!(p V !q)~~

! (p ∧ !q)

In question 3 below suppose the variable $x$ represents students and $y$ represents courses, and:

$T(x,y)$: $x$ is taking $y$.

Write the statement in good English without using variables in your answers.

3. $\exists x \forall y T(x,y)$.   See Key

4. Consider the following theorem: If $x$ is an odd integer, then $x + 2$ is odd. Give a direct proof of this theorem

See key

5. Find three subsets of {1,2,3,4,5,6,7,8,9} such that the intersection of any two has size 2 and the intersection of all three has size 1. HINT: Use a Venn Diagram.

Chapter 2

6. Suppose $U = \{1, 2, \ldots, 9\}$, $A$ = all multiples of 2, $B$ = all multiples of 3, and $C = \{3, 4, 5, 6, 7\}$. Find $C - (B - A)$.

Chapter 2

7. Give an example of a function $f: \mathbf{R} \rightarrow \mathbf{R}$ that is Onto but not 1-1. Remember that Onto means every element in the codomain gets hit at least once, and 1-1 means that no element in the codomain gets hit more than once. So you need a function where they all get hit at least once, and at least 1 of them gets hit more than once.

Chapter 2

8. Suppose Set A has 5 elements and set B has 3 elements. How many different functions map the elements of A onto the elements of B?

Chapter 2

9. Find a formula that generates the following sequence: 15,20,25,30,35,....

$a_n =$ Chapter 2

10. List all positive integers between 2 and 30 that are relatively prime to 20.

20 is made of 2 and 5

There fore, 3, 7, 9, 11, 13, 17, 19, 21, 23, 27, 29

11. Find the hexadecimal representation of $ABC_{16} + 2F5_{16}$.

$$\begin{array}{r} {}^{1}A\,{}^{1}B\,C \\ +\ 2\ F\ 5 \\ \hline D\ B\ 1_{16} \end{array}$$

12. What sequence of pseudorandom numbers is generated using the linear congruential generator $x_{n+1} = (5x_n+3) \bmod 13$ with seed $x_0 = 1$? List the numbers until they start repeating.

1

$5 \cdot 1 + 3 = 8$

$(5 \cdot 8 + 3)\%13 = 4$

$(5 \cdot 4 + 3)\%13 = 10$

$(5 \cdot 10 + 3)\%13 = 1$, repeat!

13. Convert $(204)_{10}$ to base 2.

$204 = 2 \cdot 102 + 0$
$102 = 2 \cdot 51 + 0$
$51 = 2 \cdot 25 + 1$
$25 = 2 \cdot 12 + 1$
$12 = 2 \cdot 6 + 0$
$6 = 2 \cdot 3 + 0$
$3 = 2 \cdot 1 + 1$
$1 = 2 \cdot 0 + 1$ answer $= 11001100_2$

14. Use the Euclidean Algorithm to find gcd(900,140).

$900 = 140 \cdot 6 + 60$
$140 = 60 \cdot 2 + \boxed{20}$
$60 = 20 \cdot 3 + 0$

In questions 15 and 16 below suppose you have 30 books (15 novels, 10 history books, and 5 math books). Assume that all 30 books are different.

15. In how many ways can you gather a bag of three history books and seven novels to give to a friend?

$C(10,3) \cdot C(15,7)$

16. In how many ways can you put the 30 books in a row on a shelf if the novels are on the left, the math books are in the middle, and the history books are on the right?

$15! \cdot 5! \cdot 10!$

In questions 17 and 18 below assume that you have a bowl containing hard candies: 50 cherry, 50 strawberry, 40 orange, 70 lemon, and 40 pineapple. Assuming that the pieces of each flavor are identical,

17. How many handfuls of 15 are possible?

$CR(5,15) = C(19,15)$

Note that the supply of each flavor does not matter.

18. How many handfuls of 15 are possible with at least two pieces of each flavor?

10 of your 15 choices are made for you.

$CR(5,5) = C(9,5)$

19. Find the next largest permutation in lexicographic order after 3254671.

Target replaced with one just bigger (7)

3254716

20. What is the probability that a fair coin lands Heads 6 times in a row?

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

21. A group of ten women and ten men are in a room. A committee of four is chosen at random. Find the probability that the committee consists only of women?

$\frac{C(10,4)}{C(20,4)}$

## Answer Key

1.

| $p$ | $q$ | $r$ | $\neg(r \to \neg q) \vee (p \wedge \neg r)$ |
|---|---|---|---|
| T | T | T | T |
| T | T | F | T |
| T | F | T | F |
| T | F | F | T |
| F | T | T | T |
| F | T | F | F |
| F | F | T | F |
| F | F | F | F |
| | | | |

2. $\neg(p \wedge \neg q)$.
3. Some student is taking every course.
4. Let $x = 2k + 1$. Therefore $x + 2 = 2k + 1 + 2 = 2(k + 1) + 1$, which is odd.
5. For example, {1,2,3}, {2,3,4}, {1,3,4}.
6. {4,5,6,7}.
7. f(x) = (x)*(x-2)*(x+2), for example
8. *3*3*3*3*3*.
9. $a_n = 5(n + 2)$.
10. 1,3,7,9,11,13,17,19,21,23,27,29.
11. $(DB1)_{16}$.
12. 1,8,4,10,repeat
13. 1100 1100.
14. 20.
15. $\binom{10}{3}\binom{15}{7}$.
16. $15! \cdot 5! \cdot 10!$.
17. $\binom{19}{4}$.
18. $\binom{9}{4}$.

Name: Answers CS 3310. Fall 2020 Test B. Final Exam.

For question 1 below, suppose the variable $x$ represents people, and:

$F(x)$: $x$ is friendly, $T(x)$: $x$ is tall.

Write the statement using these predicates and any needed quantifiers.

1. All tall people are friendly. $\forall x\,(T(x) \rightarrow F(x))$ being Tall implies friendly

Note The answer is NOT $\forall x\,(T(x) \wedge F(x))$ everyone is Tall and Friendly

For question 2 below, suppose the variable $x$ represents students and $y$ represents courses, and:

$T(x,y)$: $x$ is taking $y$.

Write the statement in good English without using variables in your answers.

2. $\forall x \exists y T(x,y)$. All students are taking at least one course.

3. Multiply $(4103)_5$ and $(2230)_5$, keeping the result in base 5.

$$
\begin{array}{r}
4103_5 \\
\times\ 2230_5 \\
\hline
0000 \\
22314\phantom{0} \\
13211\phantom{00} \\
13211\phantom{000} \\
\hline
\boxed{2031024 0_5}
\end{array}
$$

4. Convert $(10000)_{10}$ to base 2.

$10000 = 2 \cdot 5000 + 0$
$5000 = 2 \cdot 2500 + 0$
$2500 = 2 \cdot 1250 + 0$
$1250 = 2 \cdot 625 + 0$
$625 = 2 \cdot 312 + 1$
$312 = 2 \cdot 156 + 0$
$156 = 2 \cdot 78 + 0$
$78 = 2 \cdot 39 + 0$
$39 = 2 \cdot 19 + 1$
$19 = 2 \cdot 9 + 1$
$9 = 2 \cdot 4 + 1$
$4 = 2 \cdot 2 + 0$
$2 = 2 \cdot 1 + 0$
$1 = 2 \cdot 0 + 1$

answer $\boxed{10011100010000}$

5. Convert $(123)_5$ to base 10.

$1 \cdot 25$ $2 \cdot 5$ $3 \cdot 1$

$\boxed{38}$

6. Use the Euclidean Algorithm to find gcd(580,50).

$580 = 50 \cdot 11 + 30$
$50 = 30 \cdot 1 + 20$
$30 = 20 \cdot 1 + \boxed{10}$
$20 = 10 \cdot 2 + 0$

7. Find the inverse of 9 (mod 32), using the algorithm from class. Show your work.

$32 = 9 \cdot 3 + 5$ $\quad 5 = 32 - 9 \cdot 3$ $\quad 1 = -9 + 2 \cdot (32 - 9 \cdot 3) = 2 \cdot 32 - 7 \cdot 9$
$9 = 5 \cdot 1 + 4$ $\quad 4 = 9 - 5$ $\quad 1 = 5 - (9 - 5) = -9 + 2 \cdot 5$ $\quad +32$
$5 = 4 \cdot 1 + 1$ $\quad 1 = 5 - 4$
$4 = 1 \cdot 4 + 0$ $\quad \boxed{25} \cdot 9$

8. What sequence of pseudorandom numbers is generated using the linear congruential generator $x_{n+1} = (3x_n+2) \bmod 13$ with seed $x_0 = 1$? List the numbers until they start repeating.

$3 \cdot 1 + 2 = 5$

$(3 \cdot 5+2) \% 13 = 4$

$(3 \cdot 4+2) \% 13 = 1$ repeat

For questions 9-10 below, suppose that a "word" is any string of exactly 7 letters from the 26 letter alphabet, with repeated letters allowed.

9. How many words are there? $26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26$

10. How many words contain exactly 2 vowels? $C(7,2) \cdot 5^2 \cdot 21^5$

where to put vowels   vowel choices   other choices

11. List the next 4 combinations in lexicographical order after "1257", when taking 4 items at a time from the set {1,2,3,4,5,6,7}.

1267
1345
1346
1347

For questions 12-14 below, suppose Baskin Robins has 31 flavors of ice cream. You are supposed to choose flavors, where order does NOT matter:

12. How many ways can you pick 5 flavors, where repetition is allowed?

$CR(31,5) = C(35,5)$

13. How many ways can you pick 10 flavors, where repetition is allowed – given the following restraints: Peppermint must be one of them, cherry must not be, at least 2 must be strawberry, and at most 2 must be vanilla?

$CR(31,10)$

$CR(31,9)$ peppermint chosen

$CR(30,9)$ no cherry

$CR(30,7)$ 2 strawberry chosen

$\boxed{CR(30,7) - CR(30,4)}$ subtract bad 3 or more vanilla

14. How many ways can you pick 5 flavors, where repetition is NOT allowed?

$C(31,5)$

15. What is the probability of being dealt "3 of a kind" when given 5 cards from a standard deck of 52 cards? For simplicity, you may include anything more than "3 of a kind" such as '4 of a kind" and "3 of one and 2 of another." Show your thinking.

$$\frac{13 \cdot 4 \cdot C(49,2)}{C(52,5)}$$

13 ranks to choose from
4 ways to get 3 of 4 suits
c(49,2) ways to fill in last 2 cards

16. Find the prime factorization of 90.

$2 \cdot 3 \cdot 3 \cdot 5$

For questions 17-18 below, suppose nine people (Ann, Ben, Cal, Dot, Ed, Fran, Gail, Hal, and Ida) are in a room. Five of them stand in a row for a picture. $P(9,5)$

17. In how many ways can this be done if a taller person must be to the left of a shorter person? (Assume all have different heights.) $C(9,5)$

Once you choose which 5, the order is fixed.

18. In how many ways can this be done if Ann and Ben refuse to be in a picture together? (That is, a given picture may contain Ann, or Ben, or neither, but never both.)

Bad is Ann and Ben both in picture $= \underset{\text{Ann}}{5} \cdot \underset{\text{Ben}}{4} \cdot \underset{\text{rest}}{p(7,3)}$

Good = TOTAL ~~Good~~ − Bad $= \boxed{P(9,5) - 5 \cdot 4 \cdot p(7,3)}$

19. When dealing with Boolean variables, there are several Boolean operators we know of, such as And, Or, Implies, Exclusive Or, Nand, Nor, etc. How many unique binary operators could we form? (That is, if we create another operator that has the same truth table values as the And operator, then it would Not be unique.)

| P | Q | operator |
|---|---|---|
| 0 | 0 | x |
| 0 | 1 | y |
| 1 | 0 | z |
| 1 | 1 | w |

The operator controls 4 boolean variables so there are $2^4 = 16$ max operators.

20. How many bit strings of length 10 contain at most two 1s?

$C(10,2) + C(10,1) + C(10,0)$

21. Add $(4133)_5$ and $(4234)_5$, keeping the result in base 5.

$$\begin{array}{r} 4133_5 \\ +\ 4234_5 \\ \hline 13422_5 \end{array}$$